An RF Circuit Model for Carbon Nanotubes

P. J. Burke

***Abstract*—We develop an RF circuit model for single walled carbon nanotubes for both dc and capacitively contacted geometries. By modeling the nanotube as a nanotransmission line with distributed kinetic and magnetic inductance as well as distributed quantum and electrostatic capacitance, we calculate the complex, frequency dependent impedance for a variety of measurement geometries. Exciting voltage waves on the nanotransmission line is equivalent to directly exciting the yet-to-be observed one dimensional plasmons, the low energy excitation of a Lüttinger liquid.**

***Index Terms*—Circuit model, impedence, Lüttinger liquid, nanotube, RF.**

I. Introduction

OUR GOAL IN this paper is to describe an RF circuit model for the effective electrical (dc to GHz to THz) properties of carbon nanotubes. While we restrict our attention to metallic single walled nanotubes (SWNTs), the general approach can be used to describe semiconducting carbon nanotubes, multiwalled carbon nanotubes, quantum wires in GaAs heterostructures [1], and any other system of one-dimensional (1-D) interacting electrons [2]. An additional goal of this paper is to describe a technique that can be used to directly excite 1-D plasmons in carbon nanotubes using a microwave signal generator. This technique was recently applied to measure collective oscillations (plasmons) in a two-dimensional electron gas, including measurements of the 2-D plasmon velocity, as well as the temperature and disorder dependent damping [3]. The high frequency circuit model developed herein may have direct applications in determining the switching speed of a variety of nanotube based electronic devices.

In our recent 2-D plasmon work, we suggested a transmission-line effective circuit model to relate our electrical impedance measurements to the properties of the 2-D plasmon collective excitation [3]–[6]. There, we measured the kinetic inductance of a two-dimensional electron gas, as well as its distributed electrostatic capacitance to a metallic "gate" by directly exciting it with a microwave voltage. The distributed capacitance and inductance form a transmission line, which is an electrical engineer's view of a 2-D plasmon.

Since then, the transmission-line description has been discussed in the context of both single-walled [7] and multiwalled [8], [9] carbon nanotubes. In reference [7], by considering the Lagrangian of a one-dimensional electron gas (1-DEG), an expression for the quantum capacitance (which was not important in our 2-D experiments) as well as the kinetic inductance of a SWNT is derived. In reference [8], [9] the tunnel conductance at high voltages is related to electrical parameters (the characteristic impedance) of the transmission line in a multiwalled nanotube. In both of these discussions, the distributed inductance and capacitance per unit length form a transmission line, which is an electrical engineer's description of a 1-D plasmon.

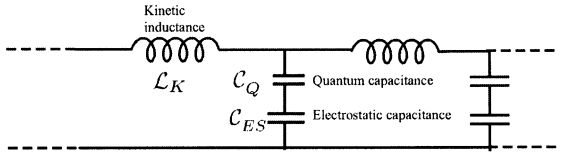

Fig. 1. Circuit diagram for 1-D system of spinless electrons. Symbols are defined per unit length.

In what follows we present an RF circuit model based on the transmission line properties of a carbon nanotube. We use this model to calculate the nanotube dynamical impedance (real and imaginary) as a function of frequency, as well as the ac damping and wave velocity. We discuss possible practical consequences [10] of the results in nanotube electronic and micromechanical/nanomechanical high-frequency circuits. A more detailed discussion of the RF circuit model can be found in our recent manuscript [11]. A description from a theoretical physics point of view (complementary to the circuit description discussed here) can be found in [12], and references therein.

II. Nano-Transmission Line

The dc circuit model for a one-channel quantum wire of noninteracting electrons is well known from the Landauer–Büttiker formalism of conduction in quantum systems. The dc conductance is simply given by e^2/h. If the spin degree of freedom is accounted for, there are two "channels" in a quantum wire: spin up and spin down, both in parallel. We postpone our discussion of spin until the next section, and assume for the moment the electrons are spinless. At ac, the circuit model is not well established experimentally. In this manuscript we discuss a transmission line equivalent circuit model that we use to predict the dynamical impedance of a SWNT under a variety of measurement geometries.

The effective circuit diagram we are proposing is shown in Fig. 1. Below, we will discuss each of the four contributions to the total circuit, and then discuss some of its general properties, such as the wave velocity and characteristic impedance. For the sake of simplicity, as shown in Fig. 2, we will restrict ourselves to the case of a wire over a "ground plane."

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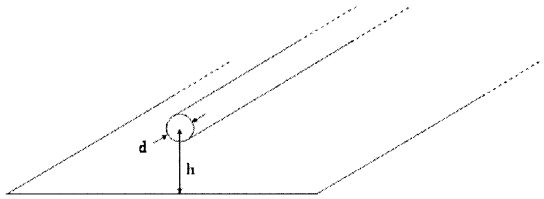

Fig. 2. Geometry of nanotube in presence of a ground plane.

A. Kinetic Inductance

In order to calculate the kinetic inductance per unit length, we follow reference [7] and calculate the kinetic energy per unit length and equate that with the $1/2LI^2$ energy of the kinetic inductance. The kinetic energy per unit length in a 1-D wire is the sum of the kinetic energies of the left movers and right movers. If there is a net current in the wire, then there are more left movers than right movers, say. If the Fermi level of the left movers is raised by $e\Delta\mu/2$, and the Fermi-level of the right movers is decreased by the same amount, then the current in the 1-D wire is $I = e^2/h\Delta\mu$. The net increase in energy of the system is the excess number of electrons ($N = e\Delta\mu/2\delta$) in the left vs. right moving states times the energy added per electron $e\Delta\mu/2$. Here δ is the single particle energy level spacing, which is related to the Fermi velocity through $\delta = \hbar v_F 2\pi/L$. Thus the excess kinetic energy is given by $hI^2/4v_F e^2$. By equating this energy with the $1/2LI^2$ energy, we have the following expression for the kinetic energy per unit length:

$$\mathcal{L}_K = \frac{h}{2e^2 v_F}. \tag{1}$$

The Fermi velocity for graphene and also carbon nanotubes is usually taken as $v_F = 8\,10^5$ m/s, so that numerically

$$\mathcal{L}_K = 16\ \mathrm{nH}/\mu\mathrm{m}. \tag{2}$$

In reference [11], we show that in 1-D systems, the kinetic inductance will always dominate the magnetic inductance. This is an important point for engineering nanoelectronics: In engineering macroscopic circuits, long thin wires are usually considered to have relatively large (magnetic) inductances. This is not the case in nanowires, where the kinetic inductance dominates.

B. Electrostatic Capacitance

The electrostatic capacitance between a wire and a ground plane as shown in Fig. 2 is given by [13]

$$\mathcal{C}_E = \frac{2\pi\epsilon}{\cosh^{-1}(2h/d)} \approx \frac{2\pi\epsilon}{\ln(h/d)} \tag{3}$$

where the approximation is good to within 1% for $h > 2 - D$. (If the distance to the ground plane becomes larger than the tube length such as in some free-standing carbon nanotubes [14], another formula for the capacitance has to be used, which involves replacing h with the length of the 1-D wire.) This can be approximated numerically as

$$\mathcal{C}_E \approx 50\,\mathrm{aF}/\mu\mathrm{m}. \tag{4}$$

This is calculated using the standard technique of setting the capacitive energy equal to the stored electrostatic energy

$$\frac{Q^2}{2C} = \frac{\epsilon}{2}\int E(x)^2\, d^3x \tag{5}$$

and using the relationship between E and Q in the geometry of interest, in this case a wire on top of a ground plane.

C. Quantum Capacitance

In a classical electron gas (in a box in 1-D, 2-D, or 3-D), to add an extra electron costs no energy. (One can add the electron with any arbitrary energy to the system.) In a quantum electron gas (in a box in 1-D, 2-D, or 3-D), due to the Pauli exclusion principle it is not possible to add an electron with energy less than the Fermi energy E_F. One must add an electron at an available quantum state above E_F. In a 1-D system of length L, the spacing between quantum states is given by:

$$\delta E = \frac{dE}{dk}\delta k = \hbar v_F \frac{2\pi}{L} \tag{6}$$

where L is the length of the system, and we have assumed a linear dispersion curve appropriate for carbon nanotubes. By equating this energy cost with an effective quantum capacitance [7], [8] with energy given by

$$\frac{e^2}{C_Q} = \delta E \tag{7}$$

one arrives at the following expression for the (quantum) capacitance per unit length [7], [8]:

$$\mathcal{C}_Q = \frac{2e^2}{hv_F} \tag{8}$$

which comes out to be numerically

$$\mathcal{C}_Q = 100\,\mathrm{aF}/\mu\mathrm{m}. \tag{9}$$

D. Wave Velocity

The wave velocity of a (any) transmission line with inductance per unit length $\mathcal{L}$ and capacitance per unit length $\mathcal{C}$ is simply $1/\sqrt{\mathcal{L}\mathcal{C}}$. In the case under consideration here, the inductance is simply the kinetic inductance; the total capacitance per unit length is given by

$$\mathcal{C}_{\mathrm{total}}^{-1} = \mathcal{C}_Q^{-1} + \mathcal{C}_{\mathrm{ES}}^{-1}. \tag{10}$$

If we were to neglect the screened coulomb interaction, it would be equivalent to neglecting the electrostatic capacitance. In that case we have

$$v_{\mathrm{noninteracting}} \approx \sqrt{\frac{1}{\mathcal{L}_K \mathcal{C}_Q}} = v_F. \tag{11}$$

One method of including the effect of electron–electron interactions in the context of the above discussion is simply to include the electrostatic capacitance as well as the quantum capacitance, so that the wave velocity is not quite exactly equal to the Fermi velocity

$$v_{\mathrm{interacting}} \approx \sqrt{\frac{1}{\mathcal{L}_K \mathcal{C}_{\mathrm{total}}}} = \sqrt{\frac{1}{\mathcal{L}_K \mathcal{C}_{\mathrm{ES}}} + \frac{1}{\mathcal{L}_K \mathcal{C}_Q}} > v_F. \tag{12}$$

The ratio of the wave (plasmon) velocity in the absence of interactions to the wave (plasmon) velocity in the presence of interactions has a special significance in the theory of Lüttinger liquids, and is denoted by the letter "g."

E. Characteristic Impedance

Another property of interest of the nanotransmission line is the characteristic impedance, defined as the ratio of the ac voltage to the ac current. This is especially important for measurement purposes. The characteristic impedance of a (any) transmission line with inductance per unit length $\mathcal{L}$ and capacitance per unit length $\mathcal{C}$ is simply $\sqrt{\mathcal{L}/C}$.

If one considers only the quantum capacitance and only the kinetic inductance, the characteristic impedance turns out to be the resistance quantum

$$Z_{c,\text{noninteracting}} = \sqrt{\frac{\mathcal{L}_K}{\mathcal{C}_Q}} = \frac{h}{2e^2} = 12.5\,\text{k}\Omega. \qquad (13)$$

If one both considers both components of the capacitance (electrostatic + quantum), then one finds

$$Z_{c,\text{interacting}} = \sqrt{\frac{\mathcal{L}_K}{\mathcal{C}_{\text{total}}}} = \sqrt{\frac{\mathcal{L}_K}{\mathcal{C}_{\text{ES}}} + \frac{\mathcal{L}_K}{\mathcal{C}_Q}} = g\frac{h}{2e^2} \qquad (14)$$

where we have inserted the definition of g.

F. Damping Mechanisms?

An important question to consider is the damping of the 1-D plasma waves. Currently very little is known theoretically or experimentally about the damping *mechanisms*. In the absence of such knowledge, we model the damping as distributed resistance along the length of the tube, with resistance per unit length $\mathcal{R}$. This model of damping of 2-D plasmons we recently measured [3] was successful in describing our experimental results, using the dc resistance to estimate the ac damping coefficient.

III. Spin–Charge Separation

A carbon nanotube, because of its band structure, has two propagating channels [15]. In addition, the electrons can be spin up or spin down. Hence, there are four channels in the Landauer–Büttiker formalism language. In this section we discuss an effective high-frequency circuit model which includes the contributions of all four channels, and makes the spin-charge separation (the hallmark of a Lüttinger liquid) clear and intuitive.

The noninteracting ac circuit model (i.e., one neglecting the electrostatic capacitance) of a single-walled carbon nanotube is fairly straightforward: One simply has four quantum channels in parallel each with its own kinetic inductance and quantum capacitance per unit length. All of the above calculations would apply to that system, accept that there are four transmission lines in parallel.

When one includes the effect of electrostatic capacitance, the four individual propagating modes all share the same capacitance to the ground plane, and hence become coupled. The equations of motion for the four coupled transmission lines can be diagonalized, and the eigenmodes become three spin waves and one charged (voltage) wave. In reference [11], we show that the circuit model of Fig. 1 is still valid as an effective circuit model for the charged mode if the kinetic inductance and quantum capacitance (but *not* the electrostatic capacitance) are simply divided by *four*.

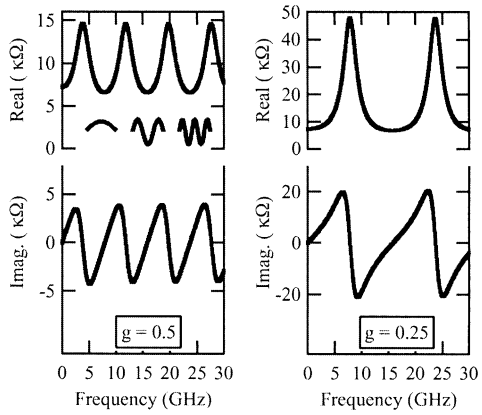

Fig. 3. Predicted nanotube dynamical impedance for ohmic contact, for two different values of g. We assume $l = 100\ \mu$m, and $\mathcal{R} = 10\ \Omega/\mu$m.

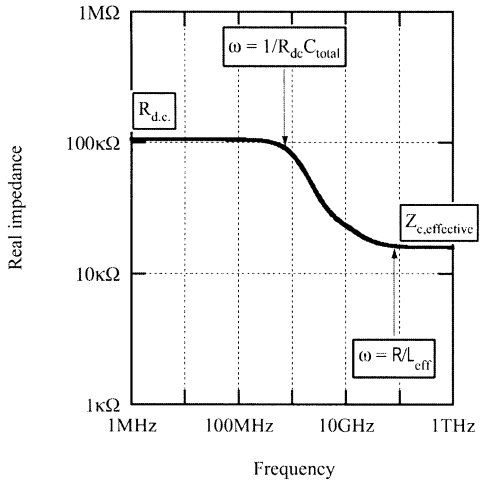

Fig. 4. Predicted nanotube dynamical impedance in overdamped case. We assume $l = 100\ \mu$m, $\mathcal{R} = 1\ \text{k}\Omega/\mu$m, and $g = 0.25$.

IV. Numerical Predictions

Using the above described circuit description, we numerically calculate the complex, frequency dependent impedance of an electrically contacted metallic single nanotube (including the effects of spin) for two cases: high damping and low damping, i.e., high and low resistance per unit length. In the low damping limit, shown in Fig. 3, the predicted impedance has resonant frequency behavior, corresponding to standing waves along the length of the tube.

In the high-damping limit, shown in Fig. 4, the resonances are washed out and one sees two limits. First, at dc the real impedance is simply the resistance per length times the length, i.e., $\mathcal{R}l$, plus the Landauer–Büttiker contact resistance. The frequency scale at which the impedance starts to change is given by the inverse of the total capacitance ($\mathcal{C}_{\text{total}}l$) times the total resistance. At very high frequencies, the impedance becomes equal to the characteristic impedance given in equation (14) (corrected

for the spin-charge effects discussed in Section III). The frequency at which this occurs is given by the inverse of the effective "LR" time constant, which is the resistance per unit length divided by the inductance per unit length.

V. Conclusion

We have derived an effective RF circuit model for a single walled carbon nanotube, including the effects of kinetic inductance as well as the electrostatic and quantum capacitance. The nanotransmission line model we developed is a circuit description of a 1-D plasmon, and as such is directly related to the long postulated Lüttinger liquid properties of 1-D systems. Our next step will be to experimentally test the validity of this circuit model in either the frequency or time domain, and to develop an RF circuit model for *active* nanotube devices, such as gated semiconducting nanotube FETs with source and drain [16].

Acknowledgment

The author thanks J. P. Eisenstein and C. Dekker for useful discussions.

References

[1] O. M. Auslaender, A. Yacoby, R. de Picciotto, K. W. Baldwin, L. N. Pfieffer, and K. W. West, "Tunneling spectroscopy of the elementary excitations in a one-dimensional wire," *Science*, vol. 295, pp. 825–828, 2002.

[2] M. P. A. Fisher and L. I. Glazman, "Transport in a one-dimensional Lüttinger liquid," in *Mesoscopic Electron Transport*. Dordrecht, The Netherlands: Kluwer, 1997.

[3] P. J. Burke, I. B. Spielman, J. P. Eisenstein, L. N. Pfeiffer, and K. W. West, "High frequency conductivity of the high-mobility two-dimensional electron gas," *Appl. Phys. Lett.*, vol. 76, no. 6, pp. 745–747, Feb. 2000.

[4] P. J. Burke and J. P. Eisenstein. (1998, Aug.) Interlayer plasmons. [Online]. Available: http://nano.ece.uci.edu

[5] X. G. Peralta, S. J. Allen, N. E. Harff, M. C. Wanke, M. P. Lilly, J. A. Simmons, J. L. Reno, P. J. Burke, and J. P. Eisenstein, "Terahertz photoconductivity and plasmon modes in double quantum well field effect transistors," *Appl. Phys. Lett.*, vol. 81, no. 9, pp. 1627–1630, Aug. 2002.

[6] X. G. Peralta, "Terahertz plasmon modes in grating coupled double quantum well field effect transistors," Ph.D. dissertation, Univ. California, Santa Barbara, CA, 2002.

[7] M. W. Bockrath, "Carbon nanotubes electrons in one dimension," Ph.D. dissertation, Univ. California, Berkeley, CA, 1999.

[8] R. Tarkiainen, M. Ahlskog, J. Pentillä, L. Roschier, P. Hakonen, M. Paalanen, and E. Sonin, "Multiwalled carbon nanotube: Lüttinger versus Fermi liquid," *Phys. Rev. B, Condens. Matter*, vol. 64, pp. 195 412-1–195 412-4, Oct. 2001.

[9] E. B. Sonin, "Tunneling into 1-D and quasi-1-D conductors and Lüttinger-liquid behavior," *J. Low Temp. Phys.*, vol. 1, pp. 321–334, 2001.

[10] M. Dyakonov and M. S. Shur, "Terahertz sources and systems," in *NATO Science Series, II. Mathematics, Physics and Chemistry*. Norwell, MA: Kluwer, 2001, vol. 27, pp. 187–207.

[11] P. J. Burke, "Luttinger Liquid Theory as a model of the gigahertz electrical properties of carbon nanotubes," *IEEE Trans. Nanotechnol.*, vol. 1, pp. 129–144, Sept. 2002.

[12] Ya. M. Blanter, F. W. J. Hekking, and M. Büttiker, "Interaction constants and dynamic conductance of a gated wire," *Phys. Rev. Lett.*, vol. 81, no. 9, pp. 1925–1928, Oct. 1998.

[13] S. Ramo, J. R. Whinnery, and T. Van Duzer, *Fields and Waves in Communication Electronics*. New York: Wiley, 1994.

[14] H. Dai, "Carbon nanotubes synthesis, structures, properties, and applications," in *Topics in Applied Physics*. Berlin, Germany: Springer, 2001, vol. 80, ch. Nanotube Growth and Characterization.

[15] Ph. Avouris, M. S. Dresselhaus, and G. Dresselhaus, Eds., "Carbon nanotubes: Synthesis, structure, properties, and applications," in *Topics in Applied Physics*. Berlin, Germany: Springer, 2001, vol. 80.

[16] S. J. Tans, A. R. M. Verschueren, and C. Dekker, "Room temperature transistor based on a single carbon nanotube," *Nature*, vol. 393, no. 49, 1998.

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Correspondence

Corrections to "An RF Circuit Model for Carbon Nanotubes"

P. J. Burke

In the above paper [1, eq. (14)], g should be replaced by g^{-1}. Thus, (14) should read

$$Z_{c,\text{interacting}} = \sqrt{\frac{\mathcal{L}_K}{\mathcal{C}_{\text{total}}}} = \sqrt{\frac{\mathcal{L}_K}{\mathcal{C}_{\text{ES}}} + \frac{\mathcal{L}_K}{\mathcal{C}_Q}} = g^{-1}\frac{h}{2e^2}.$$

The last sentence in Section III should read: In [11], we show that the circuit model of Fig. 1 is still valid as an effective circuit model for the charged mode if $\mathcal{L}_K$ is replaced by $\mathcal{L}_K/4$ and $\mathcal{C}_Q$ is replaced by $4\mathcal{C}_Q$.

References

[1] P. J. Burke, "An RF circuit model for carbon nanotubes," *IEEE Trans. Nanotechnol.*, vol. 2, pp. 55–58, Mar. 2003.

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Corrections to "Luttinger Liquid Theory as a Model of the Gigahertz Electrical Properties of Carbon Nanotubes"

P. J. Burke

In the above paper [1, p. 136], the equation appearing in the last line in the left-hand-side column should have read $(\mathcal{C}_{\text{ES}}^{-1} + (4\mathcal{C}_Q)^{-1})^{-1}$.

In the above paper [1], (43) should have read as follows:

$$\mathcal{C}_{\text{eff}}^{-1} = (4\mathcal{C}_Q)^{-1} + \mathcal{C}_{\text{ES}}^{-1}.$$

In the above paper [1], Figs. 5, 7, and 11 should have appeared as shown below in Figs. 1–3.

References

[1] P. J. Burke, "Luttinger liquid theory as a model of the gigahertz electrical properties of carbon nanotubes," *IEEE Trans. Nanotechnol.*, vol. 1, pp. 129–144, Sept. 2002.

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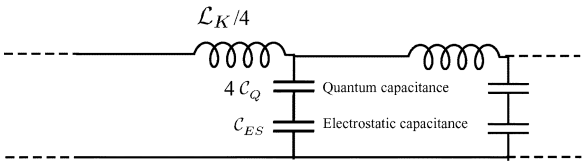

Fig. 1. Charge mode ("common mode") effective circuit diagram.

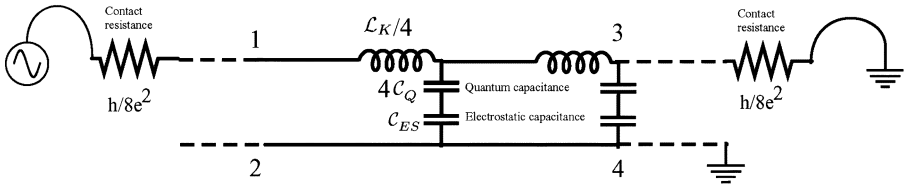

Fig. 2. *Effective* circuit diagram for an SWNT with dc electrical contacts at both ends.

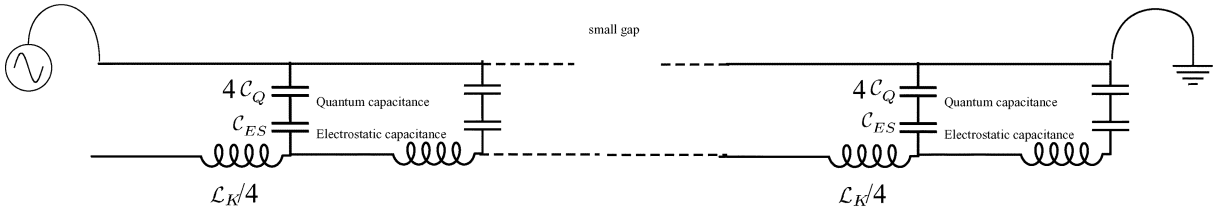

Fig. 3. Circuit diagram for capacitively coupled nanotube.