Interlayer Plasmons

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1 Introduction

This document discusses the frequency dependent electrical properties of single layer 2DEGs, 2DEGs covered by a gate, and double layer 2DEGs.

2 Single layer conductivity

The Drude formula for the frequency dependent conductivity $\sigma(\omega)$ is

$$\sigma(\omega) = \frac{ne^2\tau_{mom}}{m}\frac{1}{1+i\omega\tau_{mom}}, \tag{1}$$

with n the density, e the electron charge, m the effective mass, and τ_{mom} the momentum scattering time. This can be inverted to find the frequency dependent resistivity $\rho(\omega)$:

$$\rho(\omega) = \frac{m}{ne^2\tau_{mom}}\left(1+i\omega\tau_{mom}\right). \tag{2}$$

Numerically, for electrons in GaAs, a mobility of 10^6 $cm^2/V-s$ corresponds to a value for τ_{mom} of 38 ps; then $\omega\tau_{mom} = 1$ at f = 4 GHz. Equation 2 is the sum of a real and an imaginary term. The real term is the resistance. The imaginary term is proportional to the frequency, and hence appears as an *inductance*. The inductance is referred to as the kinetic inductance, since the inductive energy ($\frac{1}{2}LI^2$) is stored in the kinetic energy ($\frac{1}{2}mv^2$) of the electrons. This is in contrast to the more familiar magnetic inductance, where the inductive energy is stored in the magnetic field ($\int B^2d^3x$).

3 Capacitively contacted single layer

When a 2DEG is covered by a highly conducting metallic gate, the equivalent electrical circuit is shown in figure 1. Here there is a certain inductance L, capacitance C, and resistance R per unit length. This is the standard circuit diagram for transmission line theory, and waves can propagate in both directions. Note that τ_{mom} is simply given by L/R in this picture. The voltage along

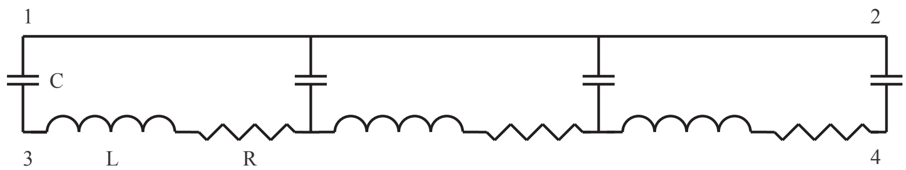

Figure 1: Capacitively contacted 2DEG.

the metal plate is constant in this picture. (If one were to include the magnetic inductance, then this would no longer be true.) The complex impedance from point 1 to point 4 Z_{14} can be determined as follows: $Z_{14} = Z_{24}$, since the metal is an equipotential. Now, Z_{24} is just the input impedance of a lossy transmission line, which is included in any textbook on microwaves or transmission lines. Thus,

$$Z_{14} = Z_c coth(\gamma l), \tag{3}$$

where l is the length of the system, Z_c the (complex) characterstic impedance defined as

$$Z_c \equiv \sqrt{\frac{R + i\omega L}{i\omega C}}, \tag{4}$$

and γ the (complex) propagation constant, defined as

$$\gamma \equiv \sqrt{(R + i\omega L)(i\omega C)}. \tag{5}$$

γ and Z_c are complex due to the dissipation. A wave travelling in one direction will decay in amplitude exponentially with a length scale given by the real part of γ. The limit of weak damping is defined in standard microwave theory as $R \ll \omega L$. This is equivalent to $\omega\tau_{mom} \gg 1$ in our case. In that limit, the decay length is approximately equal to:

$$l_{decay}^{-1} \equiv real(\gamma) = \frac{1}{2}\,\frac{R}{Z_c}. \tag{6}$$

In that limit also, $Z_c = \sqrt{L/C}$. The low frequency limit of equation 3 is:

$$\lim_{\omega\to 0}(Z_{14}) = \frac{1}{i\omega C_T} + \frac{1}{3}R_T + \frac{1}{3}i\omega L_T - \frac{1}{45}\omega R_T C_T R_T + \vartheta(\omega^2), \tag{7}$$

where R_T, L_T, and C_T are the *total* resistance, inductance, and capacitance, respectively. Thus, the dc limit is mainly a capacitor (as expected) plus some inductance and resistance.

In terms of the mobility μ, the areal density n, the width w, and the distance from the 2DEG to the metal gate d_g (where the g refers to the gate), R, L, and

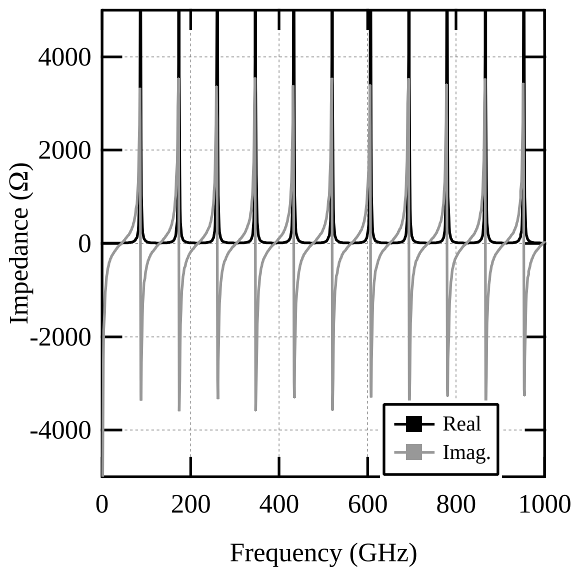

Figure 2: Impedance vs. frequency for capacitive contact. The parameters used were: l=10 μm, w=10 μm, $d_g = 5000$ $\AA$, $\mu = 2\ 10^6 cm^2/V-s$, n=1.5 $10^{11} cm^{-2}$.

C are given by:

$$R = \frac{62\ \Omega/\mu m}{n(10^{11} cm^{-2})\ \mu(10^6 cm^2/V-s)\ w(\mu m)} \tag{8}$$

$$L(nH/\mu m) = \frac{2.36\ nH/\mu m}{n(10^{11} cm^{-2})\ w(\mu m)} \tag{9}$$

$$C = 116\ 10^{-6}\ pF/\mu m\ w/d_g \tag{10}$$

The characteristic impedance, in the low damping limit, becomes:

$$Z_c = \frac{1426\ \Omega}{w(\mu m)} \sqrt{\frac{d_g(1000\ \AA)}{n(10^{11} cm^{-2})}} \tag{11}$$

The propagation constant also defines a wave velocity, and it is given by

$$v_g = \sqrt{d_g(1000\ \AA)\ n(10^{11} cm^{-2})}\ 0.6\ 10^6 m/s. \tag{12}$$

v_g refers to the wave velocity of a 2DEG under the gate. Note that this speed is typically two orders of magnitude slower than the speed of light. This is because the kinetic inductance is much larger than the magnetic inductance, slowing the wave down. Finally, the decay length can be written as

$$l_{decay} = 2 v_g \tau_{mom} = 46\ \mu m\ \sqrt{d_g(1000\ \AA)\ n(10^{11} cm^{-2})}\ \mu(10^6 cm^2/V-s) \tag{13}$$

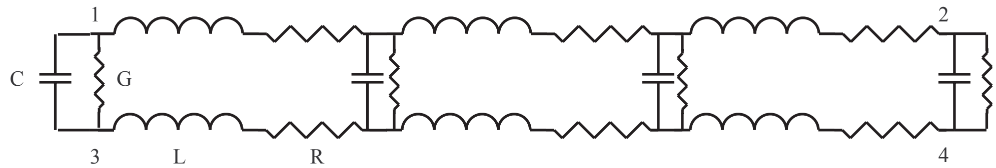

Figure 3: Double quantum well equivalent circuit.

We plot in figure 2 the contact impedance for a typical geometry, mobility, and density. Each additional peak vs. frequency corresponds to fitting one more wavelength into the effective resonator.

4 Double layer transmission line

A double well system also forms a transmission line. The equivalent circuit is given in figure 3. Below we discuss various ways of exciting this transmission line, and calculate the impedance for each of three cases. We calculate Z_{13}, Z_{14}, and Z_{34}. We have included the tunnel conductance G per length, and modeled it as a linear conductance. For the following boundary conditions, the differential equation to be solved can be cast in the following form:

$$\frac{\partial^2 V_{CM}}{\partial x^2} = 0 \tag{14}$$

$$\frac{\partial^2 V_D}{\partial x^2} - \gamma_{dw}^2 V_D = 0 \tag{15}$$

$$\gamma_{dw}^2 = 2(R + i\omega L)(G + i\omega C) \tag{16}$$

$$Z_{c\ dw} = \frac{1}{\sqrt{2}}\sqrt{\frac{R + i\omega L}{G + i\omega C}} \tag{17}$$

$$v_{dw} = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{LC}}, \tag{18}$$

where

$$V_{CM} \equiv V_{top} + V_{bottom} \tag{19}$$

$$V_D \equiv V_{top} - V_{bottom}. \tag{20}$$

Note that these differential equations include a decay length which applies for all frequencies, including dc. The dc limit cannot be taken as the low-damping limit. Rather, one finds that γ_{dw} becomes entirely real, giving a dc decay length of

$$l_{dc\ decay}^{-1} \equiv real(\gamma_{dw}) = \sqrt{2RG}. \tag{21}$$

Thus, a dc voltage difference across the layers will decay exponential in position, due to the tunneling current "shorting" out the two layers. The ac decay length for the double well system can be calculated in the weak damping limit. For non-zero tunnel conductance, the weak damping limit is defined as both $R \ll \omega L$ and $G \ll \omega C$. The first of these conditions is again equivalent to $\omega\tau_{mom} \gg 1$; the second condition is an additional constraint. If both constraints are met, the ac decay length is given by

$$l^{-1}_{ac\ decay} \equiv real(\gamma_{dw}) = \frac{1}{2}\frac{R}{Z_{c\ dw}} + G\ Z_{c\ dw}. \tag{22}$$

For photon assisted tunneling, one would like a dc equipotential, i.e. long dc decay length. The absorption of ac power occurs on a length scale given by the ac decay length, so that should be shorter than the dc decay length. The ratio of the two lengths is given by

$$\frac{l_{dc\ decay}}{l_{ac\ decay}} = \frac{1}{\sqrt{2}}\left(\frac{1}{2Z_{c\ dw}}\sqrt{\frac{R}{G}} + Z_{c\ dw}\sqrt{\frac{G}{R}}\right). \tag{23}$$

There is an interesting case, when the ac dissipation is due equally to tunneling and in plane resistance. In that case,

$$l_{ac\ decay} = l_{dc\ decay} = \frac{Z_{c\ dw}}{R} = \frac{1}{2GZ_{c\ dw}}. \tag{24}$$

In all other cases, the ac decay length is longer than the dc decay length. As the tunneling gets stronger than the critical value, the ac dissipation is mostly due to the tunnel conductance, and the decay lengths both shorten. As the tunnel strength gets weaker than the critical limit, all the decay lengths lengthen, and the ac dissipation is due mostly to the in plane resistance.

The characteristic impedance $Z_{c\ dw}$, the wave velocity v_{dw} (dw for the double well wave), and the ac decay length due to in plane resistance are given in terms of 2DEG parameters by:

$$Z_{c\ dw} = \frac{1000\ \Omega}{w(\mu m)}\sqrt{\frac{d_{dw}(1000\ \mathring{A})}{n(10^{11}cm^{-2})}} \tag{25}$$

$$v_{dw} = \sqrt{d_{dw}(1000\ \mathring{A})\ n(10^{11}cm^{-2})}\ 0.42\ 10^{6}m/s \tag{26}$$

$$l_{ac\ decay} = 2v_{dw}\tau_{mom} = 32\ \mu m\ \sqrt{d_{dw}(1000\ \mathring{A})\ n(10^{11}cm^{-2})}\ \mu(10^{6}cm^{2}/Vs) \tag{27}$$

Here n is the single layer density, d_{dw} is the distance between the double wells. There is an intesting relationship between the Fermi velocity and the wave speed:

$$\frac{v_{dw}}{v_{Fermi}} = \sqrt{\frac{d_{dw}}{a_B}}, \tag{28}$$

where a_B is the Bohr atomic radius of an electron in GaAs, $\approx$ 100 $\mathring{A}$. Therefore, the ac decay length due to in plane resistance and the mean free path are related by:

$$\frac{l_{ac\ decay}}{mfp} = \frac{2 v_{dw} \tau_{mom}}{v_{Fermi} \tau_{mom}} = 2\sqrt{\frac{a_B}{d_{dw}}}. \tag{29}$$

Samples which are less than $l_{ac\ decay}$ in length then are also in the mesoscopic regime, since the mean free path is comparable to $l_{ac\ decay}$.

The tunnel conductance also contributes to the ac decay length. We can model the tunnel conductance dependence on tunnel barrier thickness based on previous experiments. Using the experimental tunnel resistance of 250 $k\Omega$ in 250x250 μm^2 for a 175 $\AA$ barrier, and the tunnel conductance decay length of 7.2 $\AA$, we find:

$$R_{tunnel} = \frac{0.435\ \Omega}{A(\mu m^2)}\ exp\left(\frac{d_B}{7.2\ \AA}\right), \tag{30}$$

where A is the area, and d_B is the tunnel barrier thickness. This is a different quantity than d_{dw}, which is the distance which determines the capacitance. (We use the center-to-center distance for d_{dw}.) Thus, G can be written as:

$$G = 2.3\ mho/\mu m\ w(\mu m)\ exp\left(\frac{-d_B}{7.2\ \AA}\right) \tag{31}$$

Thus, the dc decay length (21) can be written as:

$$l_{dc\ decay} = 0.059\mu m\ \sqrt{n(10^{11} cm^{-2})\ \mu(10^6 cm^2/V-s)}\ exp\left(\frac{d_B}{14.4\ \AA}\right). \tag{32}$$

Thus, for an 85 $\AA$ tunnel barrier with n=2.5 $10^{11} cm^{-2}$ and μ=2 $10^6 cm^2/V-s$, we get $l_{dc\ decay} = 48\ \mu m$. The ac decay is given by

$$\begin{aligned} l^{-1}_{ac\ decay} &= \left(32\ \mu m\ \sqrt{d_{dw}(1000\ \AA)\ n(10^{11} cm^{-2})}\ \mu(10^6 cm^2/V-s)\right)^{-1} \\ &+ \left(4.35\ 10^{-4}\ \mu m\ exp(\frac{d}{7.2\ \AA})\sqrt{\frac{n(10^{11} cm^{-2})}{d_{dw}(1000\ \AA)}}\right)^{-1} \end{aligned} \tag{33}$$

In the crossover case, we have:

$$l_{ac\ decay} = l_{dc\ decay} = 16\ \mu m\ \mu(10^6 cm^2/V-s)\ \sqrt{d_{dw}(1000\ \AA)\ n(10^{11} cm^{-2})}. \tag{34}$$

The condition for the crossover is:

$$d_{dw}(1000\ \AA)\ \mu(10^6 cm/V-s)\ exp(\frac{-d_B}{7.2\ \AA}) = 13.48\ 10^{-6}. \tag{35}$$

For d=400 $\AA$ and $\mu = 2\ 10^6 cm^s/V-s$, we need $d_B = 80\ \AA$.

4.1 Boundary condition one

In this section, we calculate Z_{13}. This is very simple, since we are only exciting the differential mode. Thus, the impedance is given by equation 3, with the appropriate redefinition of the propagation constant γ_{dw} and characteristic impedance $Z_{c\ dw}$:

$$Z_{13} = Z_{c\ dw} coth(\gamma_{dw} l). \tag{36}$$

For impedance matching to antennas, this is good. The source impedance is usually of order $Z_0/\sqrt{\epsilon_{avg}}$ ($=$ 105 Ω for Si), where ϵ_{avg} is the average dielectric constant of vacuum and the dielectric lens, and $Z_0 = 377\ \Omega$ is the characteristic impedance of free space. For example, the source impedance of self-complimentary antennas (which have broadband frequency response) is 114 Ω on quartz. In order to achieve maximum coupling between the device and the incoming beam, the device impedance should be equal to the complex conjugate of the source impedance. *Broadband matching to the device is easiest if the device impedance is real and of order* 100 Ω. If $l > l_{ac\ decay}$, then equation 36 predicts the impedance seen by the antenna is simply $Z_{c\ dw}$ (since $coth(x) \to 1$ for large x), which can easily be made 100 Ω simply by adjusting the width to be about 10 μm. A wave launched onto terminals 1,3 will simply propagate until it has decayed away, and never come back. The challenge is to connect an antenna to point 1,3 in figure 3.

4.2 Boundary condition two

In this section, we calculate Z_{12}. This is a little complicated, and requires solving for the position dependent voltage and current on the line for the boundary condition of a generator at point 1 with point 2 grounded. The ratio of the voltage to the current at point 1 defines the impedance, and we find:

$$Z_{12} = \frac{1}{2} l(i\omega L + R) + Z_{c\ dw} tanh\left(\frac{\gamma_{dw} l}{2}\right). \tag{37}$$

In the limit $\omega \to 0$, we find $Z_{12} \to l(i\omega L + R)$, as expected. In that limit, the impedance becomes that of the single layer, since the other layer is isolated at dc. If there is no dissipation, than Z_{12} will have an average rise with frequency, modulated by a strongly frequency dependent term. When there is dissipation, the strength of the resonant modulation gets damped, depending on the ratio of the length of the double well system to the ac decay length. In figure 4, we plot the impedance vs. frequency for a representative geometry, mobility, and density. One sees that the high frequency limit is essentially that of an inductor. Each additional peak with increasing frequency corresponds to fitting one more wavelength into the resonator.

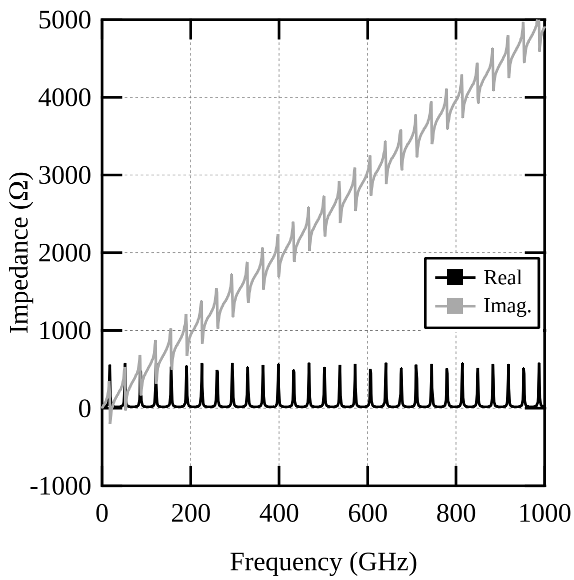

Figure 4: Impedance vs. frequency, point 1 to point 2. The parameters used were: l=10 μm, w=10 μm, $d_{dw} = 400$ $\mathring{A}$, $\mu = 2\ 10^6 cm^2/V - s$, n=1.5 $10^{11} cm^{-2}$, no tunneling.

4.3 Boundary condition three

The calculation of Z_{14} is similar to that in the previous section. We find:

$$Z_{14} = \frac{1}{2} l(i\omega L + R) + Z_{c\ dw} coth\left(\frac{\gamma_{dw} l}{2}\right). \tag{38}$$

The dc limit is more complicated than a capacitor, but the imaginary part tends to $-\infty$, as expected. The high frequency limit is again an inductor. A representative plot is given in figure 5.

5 More complicated geometries

All of the above considerations can be used to connect more complicated systems in networks. For systems smaller than the wavelength of light, the impedance of a network of elements in a plane adds according to standard circuit theory. *However*, "stacking" elements such as putting a gate on top of or below (or both!) of a double layer system will change the effective wave equation and boundary conditions, and the above calculations will not apply.

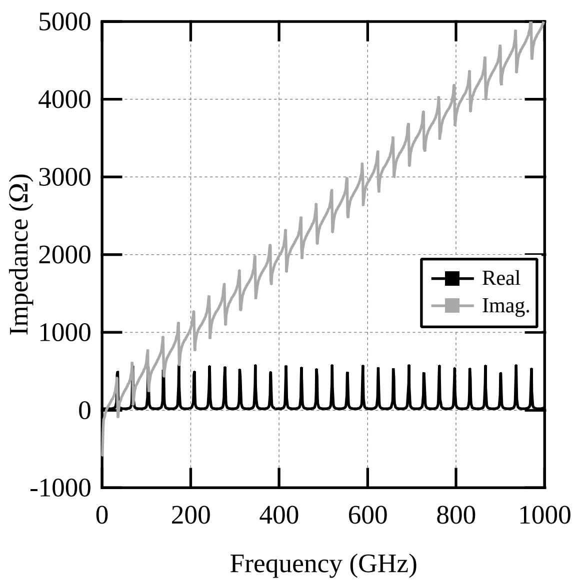

Figure 5: Impedance vs. frequency, point 1 to point 4. The parameters used were: l=10 μm, w=10 μm, $d_{dw} = 400$ Å, $\mu = 2\ 10^6 cm^2/V-s$, n=1.5 $10^{11} cm^{-2}$, no tunneling.